Numerical solutions for network simulation model of free convective flow past a vertical plate

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Abstract— The numerical study of laminar free convective flow past a viscous incompressible fluid past a vertical plate with heat and mass transfer is considered. The governing boundary layer equations of continuity, Momentum, Energy and concentration are transformed into non-dimensional form by using the non-dimensional quantities and are solved by using Network Simulation Method. The velocity, temperature, and concentration profiles have been studied for various parameters such as Schmidt number Sc , Prandtl number $\Pr$, buoyancy ratio parameter N are discussed and analyzed graphically.

Keywords: Heat and Mass transfer, Natural convection, Net work Simulation Model, PSPICE, vertical plate.

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1 Introduction

Natural convection flows under the influence of gravitational force have been investigated most extensively because they occur frequently in nature as well as in science and engineering applications. When a heated surface is in contact with the fluid, the result of temperature difference causes buoyancy force, which induces the natural convection. The transient natural convection flows over vertical bodies have a wide range of applications in engineering and technology.The study of convection with both heat and mass transfer is very useful in various fields such as industry, agriculture, and oceanography, Design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, and pollution of the environment. The analytical method fails to solve the problem of unsteady two-dimensional natural convection flow past a semi-infinite vertical plate. The advent of advanced numerical methods and the developments in computer technology pave the way to solve such difficult problems. Finite difference methods play an important role in solving the partial differential equations.

Stokes [1] first presented an exact solution to the Navier-Stokes equation which is for the flow of a viscous incompressible fluid past an impulsively started semi-infinite horizontal plate by finite - difference method of a mixed explicit-implicit type, which is convergent and stable and hence it is free from any restrictions on the mesh-size. The problem of transient free convective flow past a semi-infinite vertical isothermal plate was first studied by Siegel [2]. Later, Hellums and Churchill [3], Callahan and Marner [4] examined the problem of free convective flow past a vertical plate using an explicit finite difference scheme. Soundalgekar [5] was the first to present an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. Soundalgekar and Ganesan [6] investigated the finite difference of transient free convection with mass transfer of an isothermal vertical flat plate.The solution was derived by the Laplace transform technique and the effects of heating or cooling of the plate on the flow-field were discussed through Grashof number. .Later, Soundalgekar and Ganesan [7] solved the free convection flow of an isothermal flat plate using implicit finite difference method. Chen et al. [8] studied the flows and heat transfer characteristics of laminar free convection in boundary layer flows from horizontal, inclined and vertical plates with variable wall temperature and heat flux. Ganesan and Palani [9] considered the effects of MHD on a two-dimensional freeconvectiveflow of a viscous incompressible fluid past a semi-infinite isothermal vertical plate using an efficient implicit finite difference method. Here, the dimensionless governing equations are unsteady, two-dimensional, coupled,Non-linear integro-differential equations which are solved numerically using an implicit finite difference scheme.

Many authors use the finite difference method to solve the free convective flow past method .The present problem the free convective flow past a vertical plate with heat and mass transfer is solved using a new method called the Network Simulation Method (NSM). NSM simulates the behaviour of unsteady electric circuits by means of resistors, capacitors and non–linear devices that seek to resemble thermal systems governed by unsteady linear or non–linear equations. Electrical, Thermal motion analogy provides a network model that is solved by means of a very common program used to simulate electrical circuits, PSPICE [10].

NSM yields the ordinary differential equations which are the basis for implementing standard electrical network model for an elemental control volume from the partial differential equations that define the mathematical model of physical process and by means of spatial discretization [11]. Alhama [12] represents the relation between NSM and heat transfer. Alhama et al.[13] studied the numerical solution of the heat conduction equation with the electro – thermal analogy and the code PSPICE.Zueco [14] studied for dissipative fluid, free convective flow past a vertical plate with constant heat flux using NSM.The effects of thermal radiation and viscous dissipation on magneto-hydrodynamic (MHD) unsteady free-convectionflow over a semi-infinite vertical porous plate are analysed by Zueco[15]. The fluid considered is non-gray (absorption coefficient dependenton wave length). The Network Simulation Method is used to solve the boundary-layer equations based on the finitedifferenceformulation.The two-dimensional unsteady channel convection with viscous heating effects was studied by Zueco [16].The steady, laminar axi-symmetric convective heat and mass transfer in boundary layer flow over a vertical thin cylindrical configuration in the presence of significant surface heat and mass flux is studied theoretically and numerically by Zueco et al. [17]. Zueco et al.[18] studied the conjugated heat transfer problem for laminar flows in pipes(including bi-dimensional wall and axial conduction in the fluid) is studied both for the transient and steady states and solved using the network simulation method.

2. Mathematical Analysis

A two dimensional laminar free convective flow viscous incompressiblefluid past a semi-infinite vertical plate is considered. Assume that x axis is taken along the plate, and the y axis is taken along the normal to the plate(as shown in Fig.1). It is assumed that the concentration C' of the diffusing species in a binary mixture is very low in comparison to the other. This leads to the assumption that the Soret and Dufour effects are negligible. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and there is no chemical reaction between the fluid and diffusing species. Initially, it is assumed that the plate and the fluid are at the same temperature T'_{∞} and concentration C'_{∞}. As the time increases, the plate temperature and the level of the species concentration are raised to T'_{w} and C'_{w} respectively.

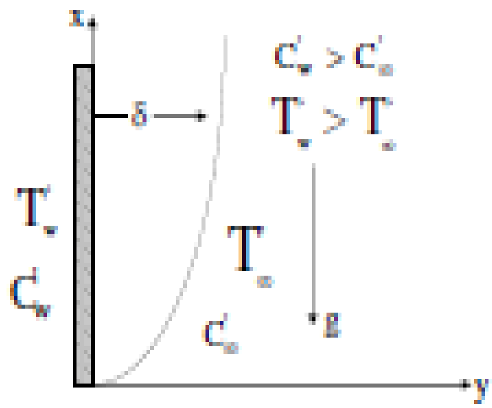

Fig. 1. The physical coordinate system

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Under these assumptions and incorporating the Boussinesq approximation within the boundary layer, the governing equations of continuity, momentum and energy respectively are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T'-T'_{\infty})+g\beta_c(C'-C'_{\infty})+\upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2} \tag{4}$$

The initial and boundary conditions are

$$\begin{array}{llll} t'\le 0: u=0, & v=0, & T'=T'_{\infty}, & C'=C'_{\infty} \text{ for all } x \text{ and } y \\ t'>0: u=0, & v=0, & T'=T'_{w}, & C'=C'_{w} \quad at\ \ y=0 \\ \quad u=0, & & T'=T'_{\infty}, & C'=C'_{\infty} \quad at\ \ x=0 \\ \quad u\to 0, & & T'\to T'_{\infty}, & C'\to C'_{\infty} \quad as\ \ y\to\infty \end{array} \tag{5}$$

Using the following non-dimensional quantities:

$$X=\frac{x}{L},\ Y=\frac{y}{L}(Gr_L)^{\frac{1}{4}}, V=\frac{vL}{\upsilon}(Gr_L)^{\frac{-1}{4}}, U=\frac{uL}{\upsilon}(Gr_L)^{\frac{-1}{2}}$$

$$t=\frac{\upsilon t'}{L^2}(Gr_L)^{\frac{1}{2}}, T=\frac{(T'-T'_{\infty})}{(T'_w-T'_{\infty})}, C=\frac{(C'-C'_{\infty})}{(C'_w-C'_{\infty})}, N=\frac{Gr^*}{Gr_L}, \tag{6}$$

$$Gr_L=\frac{g\beta(T'_w-T'_{\infty})L^3}{\upsilon^2}, Gr^*=\frac{g\beta_c(C'_w-C'_{\infty})L^3}{\upsilon^2}, \Pr=\frac{\upsilon}{\alpha}, Sc=\frac{\upsilon}{D},$$

Equations (1), (2), (3), (4) and (5) can then be written in the following non-dimensional form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{7}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=T+NC+\frac{\partial^2 U}{\partial Y^2} \tag{8}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2} \tag{9}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{10}$$

The corresponding non-dimensional initial and boundary conditions are

$$\begin{array}{llll} t\le 0: U=0, V=0, & T=0, C=0 & \text{for all } x \text{ and } y \\ t>0: U=0, V=0, & T=1, C=1 & \text{at } Y=0 \\ \quad U=0, & T=0,\ C=0 & \text{at } X=0 \\ \quad U\to 0, & T\to 0, C\to 0 & \text{at } X=0 \end{array} \tag{11}$$

3. Solution Procedure

The governing partial differential equations (7)–(10) are unsteady, coupled and non-linear with initial and derivative boundary conditions (11). They are solved numerically by Network Simulation Method (NSM) described in detail by [12] – [18] using [10] and [11]. Also network model designed (combinations of resistors R_u & R_T, current control generators G_u & G_T and capacitors C_u & C_T) as shown in Fig. 1a-1c is explained by Zueco et al. [17, 18] and Zueco [14,15,16]. Finally it is worth to satisfy Kirchhoff's law.

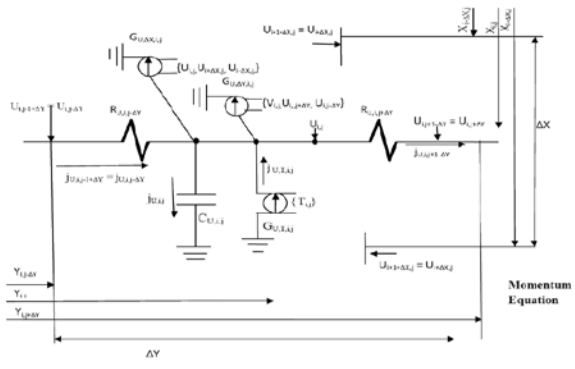

Fig.1a : Network model of the control volume- momentum equation

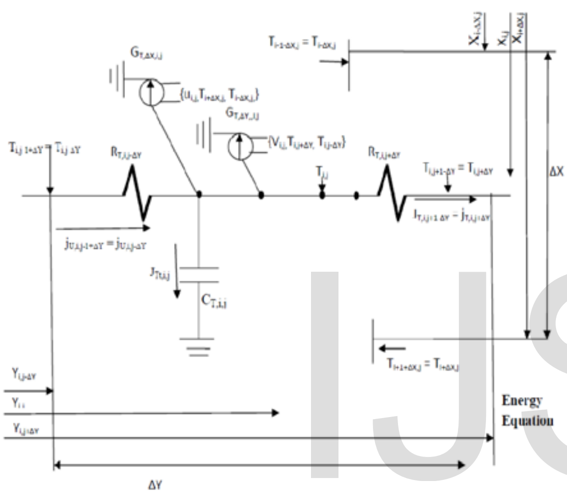

Fig.1b : Network model of the control volume- energy equation

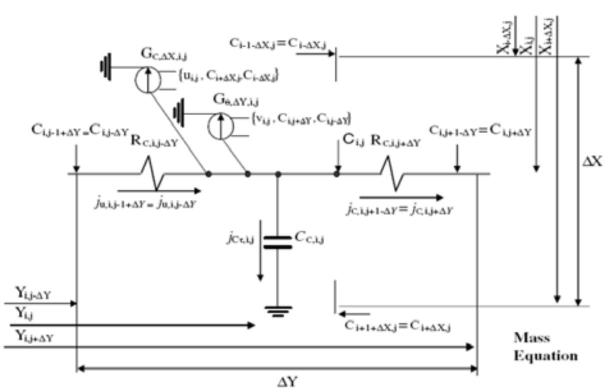

Fig. 1c: Network model of the control volume-Mass equation

4. Results and Discussion:

The velocity, temperature and concentration profiles are studied for various values of Prandtl number $\Pr$, Schmidt number, Sc surface concentration power law exponent m, surface temperature power law exponent n, and the buoyancy ratio parameter N. The values of t with star (*) symbols denote the time taken to reach steady state.

Velocity, temperature and concentration profiles for different values of Prandtl number $\Pr$, are shown at the upper edge of the plate i.e., at $X=1.0$.

Fig. 2 and 3 shows that the fluid velocity and temperature increases for lower values of $\Pr$, the thermal boundary layer decreases or the larger value of Pr.

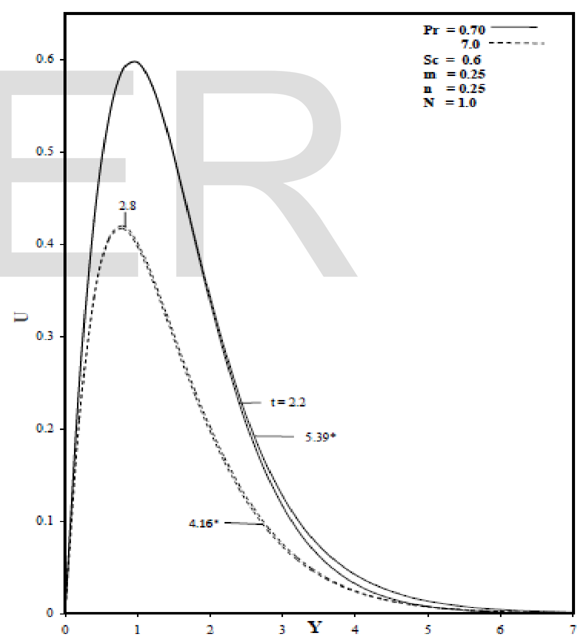

Fig. 2 : Transient Velosity profiles at X=1.0 for different values of Pr

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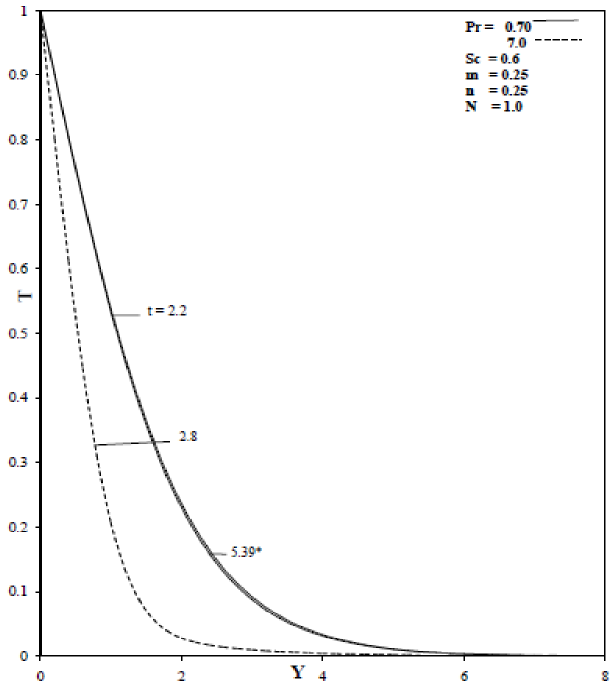

Fig. 3: Transient Temperature profiles at X=1.0 for different values of Pr

From Fig. 4 it is seen that the concentration decreases when Pr increases. For increasing Pr the time taken to reach the steady state is also increased.

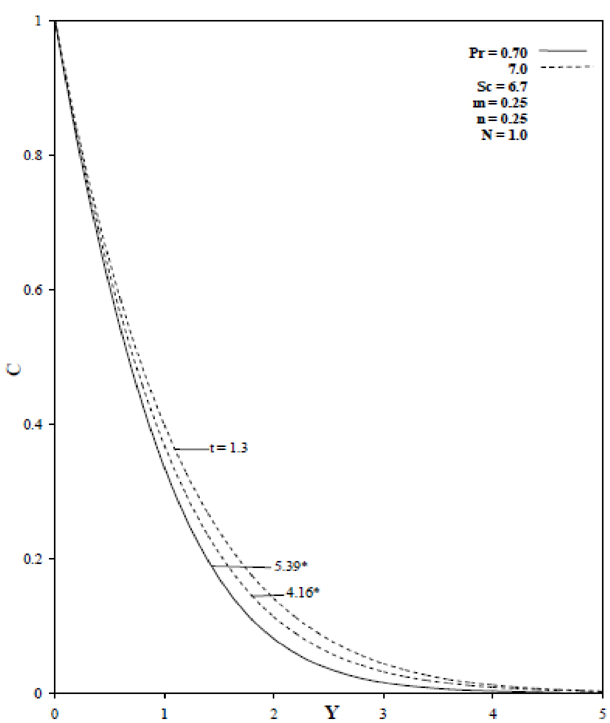

Fig. 4: Transient Concentration profiles at X=1.0 for different values of Pr

Figs. 5-7 depict the transient velocity, temperature and concentration profiles for various values of Schmidt number Sc. Fig. 5 shows that the velocity decreases with an increase in the Schmidt number Sc. Also, the boundary layer thickness decreases with an increase in Schmidt number Sc. Fig. 6 indicates that the temperature increases for larger values of Sc.

From Fig. 7 it is observed that the Schmidt number increases while the concentration decreases. This causes the concentration buoyancy effects to decrease.

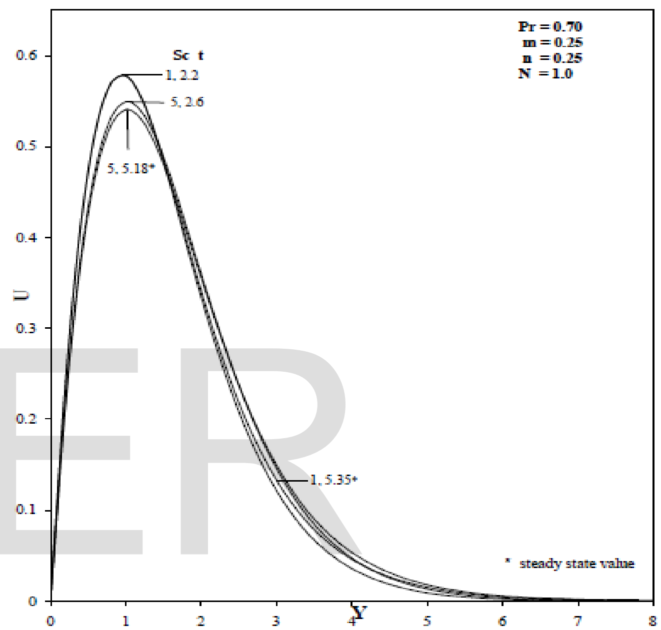

Fig. 5: Transient Velocity Profiles at X=1.0 for different values of Sc

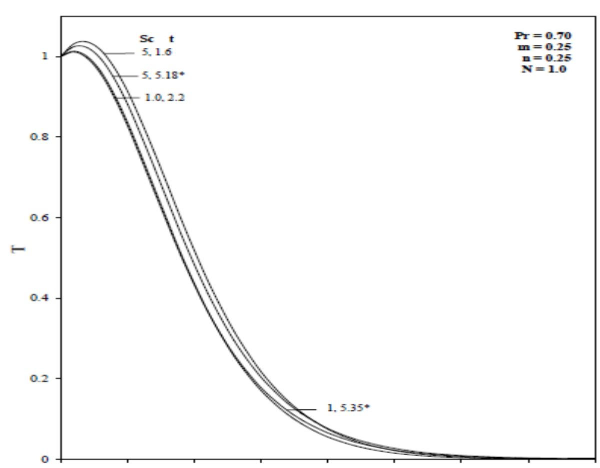

Fig. 6: Transient Temperature Profiles at X=1.0 for different values of Sc

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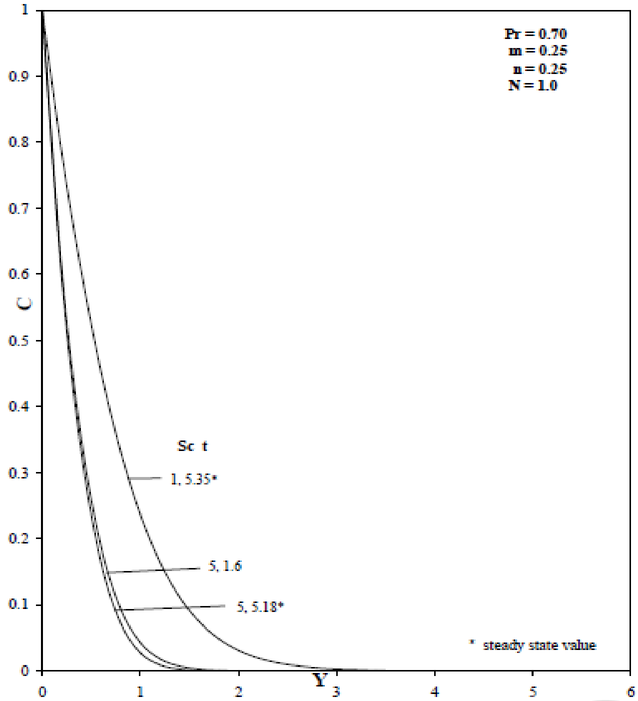

Fig. 7: Transient Concentration Profiles at X=1.0 for different values of Sc

Through Figs. 8-10 the effects of the surface concentration power law exponent m were plotted on velocity, temperature and concentration distributions. Figs. 8, 9 and 10 indicates the velocity, temperature and concentration is maximized throughout the boundary layer with a decrease in m .

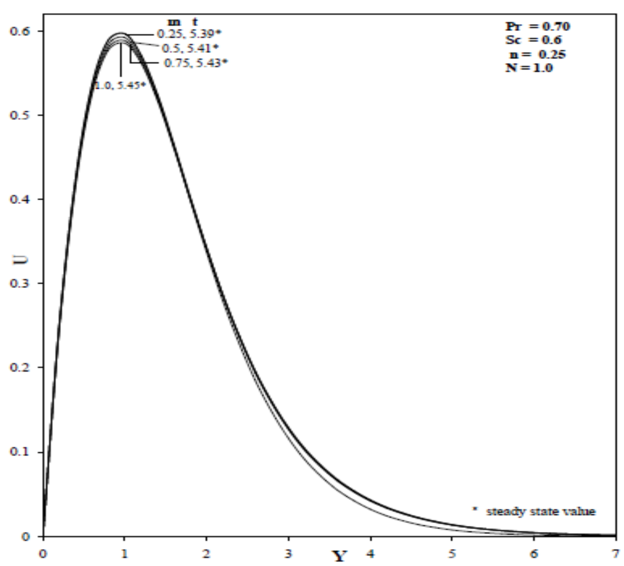

Fig. 8: Transient Velocity Profiles at X=1.0 for different values of m

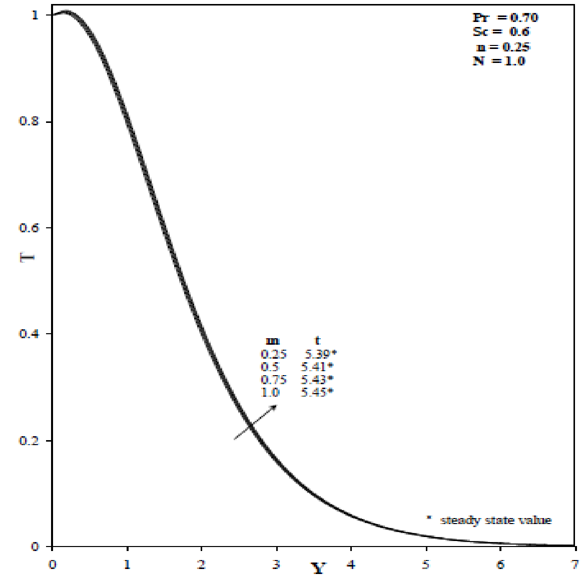

Fig. 9: Transient Temperature Profiles at X=1.0 for different values of m

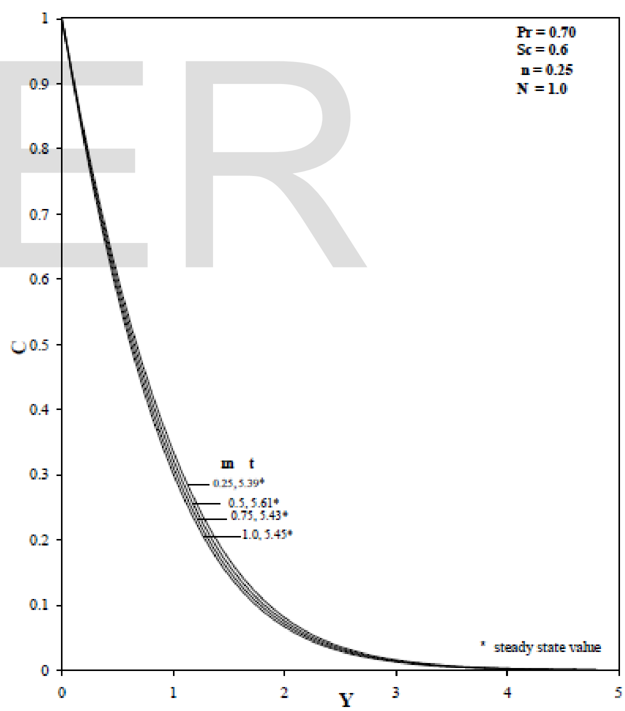

Fig 10: Transient Concentration Profiles at X=1.0 for different values of m

It is observed from Figs. 11-13 the effects of the surface temperature power law exponent n , Fig. 11 and 12 indicates that the velocity and temperature increased for smaller values of n . Fig. 13 shows that the Concentration of the species increases for larger values of n .

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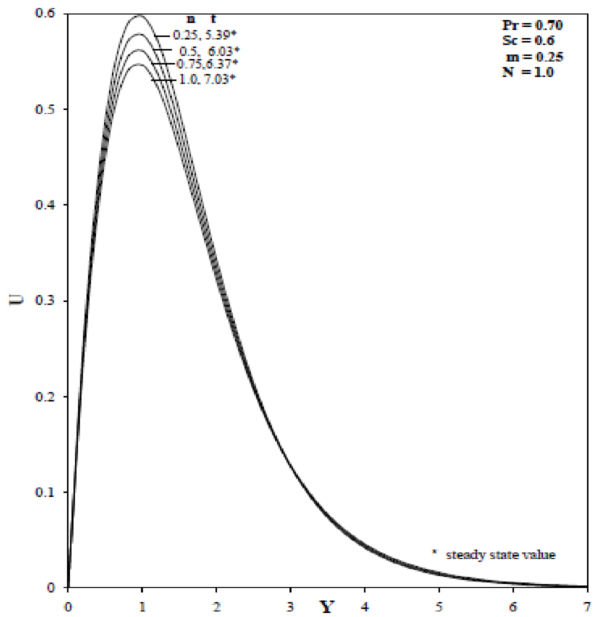

Fig. 11: Transient Velocity Profile at X=1.0 for different values of n

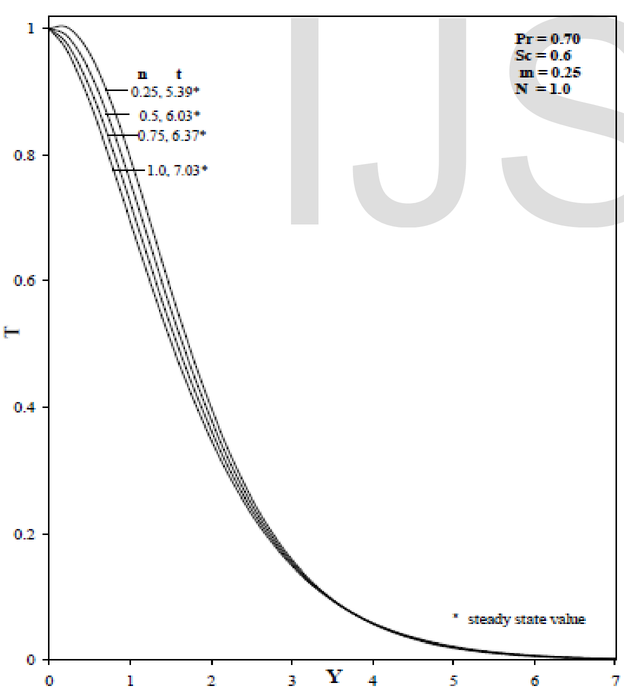

Fig 12: Transient Temperature Profile at X=1.0 for different values of n

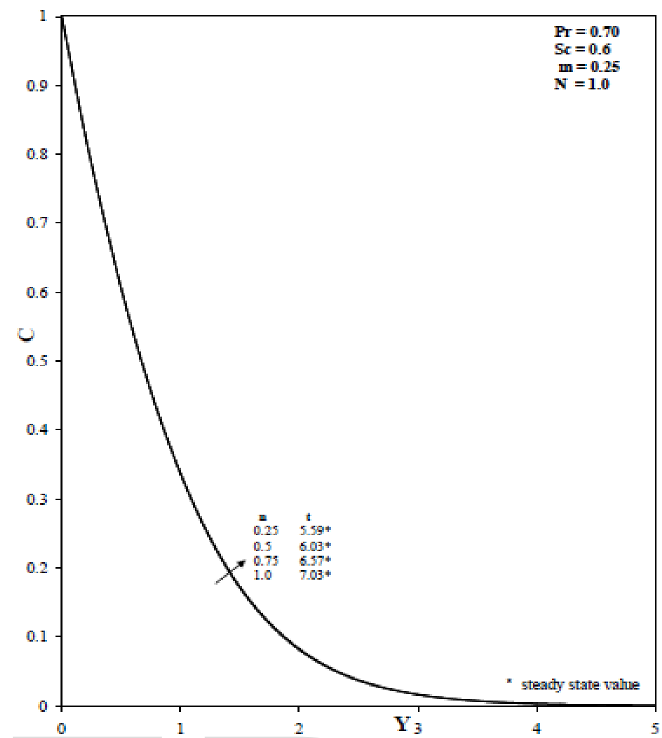

Fig. 13: Transient Concentration Profile at X=1.0 for different values of n

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It is noted from Figs. 14-16 the effects of the buoyancy ratio parameter N on the transient velocity, temperature and concentration profiles. Fig. 14 indicates an in increase in N leads to an increase in the velocity, i.e., as N increases, the combined buoyancy force also increases therefore, the velocity increases near the surface of the plate. Fig. 15 depicts the temperature decreases for all the values of N. Fig. 16 indicates for higher value of buoyancy ratio parameter N the fluid cools rapidly and concentration field decreases with increasing value of buoyancy ratio parameter N.

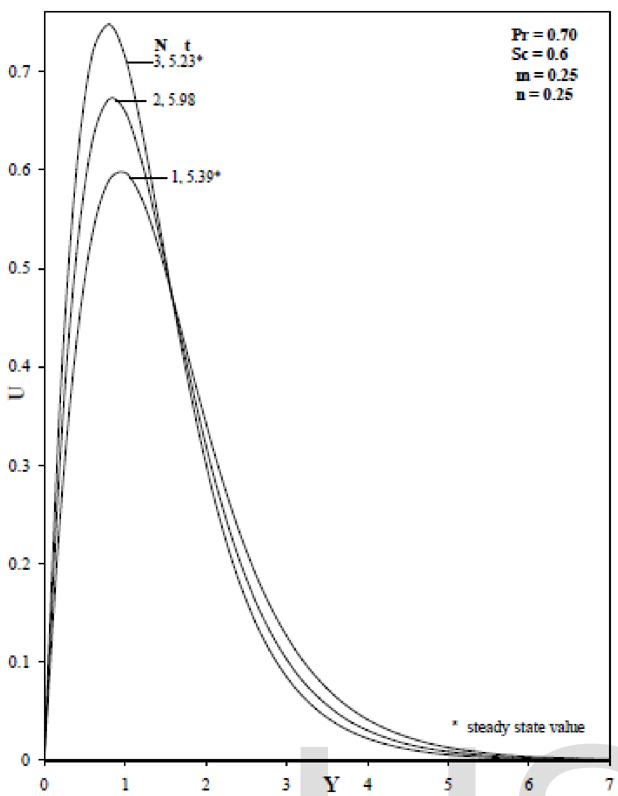

Fig. 14: Transient Velocity Profiles at X=1.0 for different values of **N**

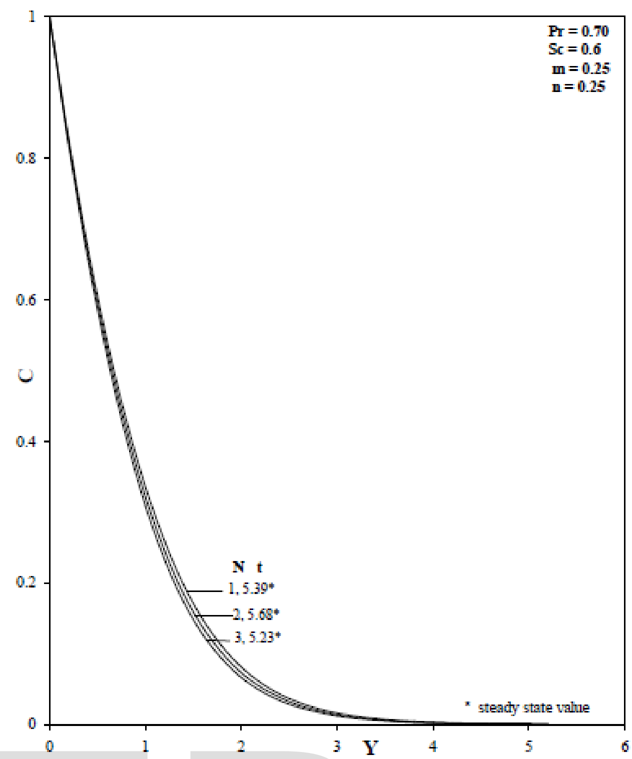

Fig.16: Transient Concentration Profiles at X=1.0 for different values of **N**

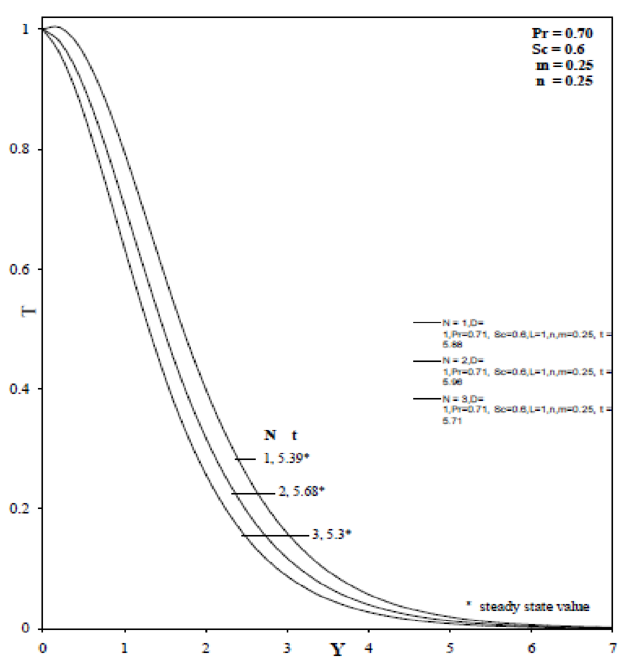

Fig. 15: Transient Temperature Profiles at X=1.0 for different values of **N**

4. Conclusions

A mathematical model has been presented for the free convection flow from a vertical plate with non-isothermal surface temperature and concentration. The family of governing partial differential equations is solved by an efficient NSM technique and parametric study is performed to illustrate the influence of thermo physical parameters on the velocity, temperature and concentration profiles. It has been shown that:

- The time taken to reach steady state increases with increasing $\Pr$, Sc, m, n and N.
- The fluid velocity increases for higher values of N and lower values of $\Pr$, Sc, m and n.
- Temperature increases for larger values of , Sc smaller values of $\Pr$, m, n and N.
- Concentration of species decreases for larger values of $\Pr$, Sc, m and N and smaller values of n.

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